Scope International Journal of Science, Humanities, Management and Technology. ISSN : 2455-068X
Vol.4 Issue 3 (2018) 31 - 36. Submitted 20/06/2018. Published 16/07/2018

ON THE LEVEL SPACES OF FUZZY TOPOLOGICAL SPACES

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ABSTRACT

The thesis deals mainly with fuzzy topological spaces, intuitionistic fuzzy sets, intuitionistic fuzzy topological spaces, fuzzy numbers, intuitionistic fuzzy numbers, interval-valued intuitionistic fuzzy numbers and applications of fuzzy mathematics. It is the purpose of this paper to go somewhat deeper into the structure of fuzzy topological spaces. In doing so we found we had to alter the definition of a fuzzy topology used up to now. We shall also introduce two functors \gw and \gi which will allow us to see more clearly the connection between fuzzy topological spaces and topological spaces.

PRELIMINARIES

In this chapter, the preliminary definitions which are all needful further discussion of our dissertation.

Definition

A **topology** on a set X is a collection τ of subsets X having the following properties

i) ϕ and X are in τ

ii) The union of the elements of any sub collection of τ is in τ

iii) The intersection of the elements of any finite sub collection of τ is in τ .

Example

Consider a set X = {a, b, c}

Then τ ={X, ϕ, (a), (b), (a, b), (a, c)} is a topology.

Definition

Let X be a non-empty set the symbol I will denote the unit interval [0,1] a **fuzzy set** in X is a function with domain X and values in I that is an element of I.

Definition

A set X for which a topology τ has been specified is called a **topological space.**

Definition

A **fuzzy topology** is a family τ of fuzzy sets in X which satisfies the following conditions:

i) ϕ and x are in $\tau, \phi, X \in \tau$

ii) If $A, B \in \tau$ then $A \cap B \in \tau$

iii) If $A_i \in \tau$ for each $i \in I$ then $U_{i \in I} A_i \in \tau$ then τ is called a fuzzy topology.

Definition

The pair (X, τ) is called the **fuzzy topology space**.

Definition

A **fuzzy point** is a is fuzzy set p_x which takes the value 0 for all $y \in X$ except one, that is $x \in A$ denoted by $p_x \in A$ iff $p_x(x) \in A(x)$.

Definition

Let (X, τ) be a fuzzy topological spaces. We say x is **interpreservative** if the intersection each family of open sets is an open sets or equivalently if family of closed sets is a fuzzy topology on X. we say X is **locally minimal** if $\cap\{G \in \tau : x \in G\}$ is open for each $x \in X$ that is each $x \in X$ admits a smallest neighbourhood we say X is α **-locally minimal** $\alpha \in [0,1]$ if $\cap\{G \in \tau : x \in G\}$ is α -open, for each $x \in X$.

Definition

The fuzzy set A of X is an α -set if $\alpha(A) \geq \alpha$, moreover if A was open (closed) we will say A is α **-open (**α **-closed)**.

Definition

A set X for which a topology τ has been specified is called a **topological space.**

ON THE LEVEL SPACES OF FUZZY TOPOLOGICAL SPACES

In this chapter, we have presented about the theorems on the level spaces of fuzzy topological space.

Definition

$0 \leq \alpha < 1$ fuzzy topological space (X,T) is said to be **α-Hausdorff** (resp. α*-Hausdorff) if for each x, y in X with x not equal to y, there exist G, H in T such that G(x)>α (resp. $G(x) \geq \alpha, H(y) > \alpha$ (resp. $H(y) \geq \alpha$) and $G \wedge H = 0$.

Definition

Let $0 \leq \alpha < 1$. A fuzzy topological space (X,T) is said to be **α-Hausdorff** (resp. α*-Hausdorff) if for each x, y in X with x not equal to y, there exist G, H in T such that G(x)>α (resp. $G(x) \geq \alpha, H(y) > \alpha$ (resp. $H(y) \geq \alpha$) and G, H are α-disjoint (resp. α*-disjoint).

Definition

Let $0 \leq \alpha < 1$ $(0 \leq \alpha < 1)$. A fuzzy topological space (X,T) is said to be **α-disconnected** (resp. α*-disconnected) is there exists an α-shading (resp. α*-shading) family of two open fuzzy sets in X which are non-empty of order α (resp. order α*) and α-disjoint (resp. α*-disjoint).

Definition

Let $0 \leq \alpha < 1$ $(0 \leq \alpha < 1)$. A fuzzy topological space (X,T) is said to be **α-connected** (resp. α*-connected) is there exists an α-shading (resp. α*-shading) family of two open fuzzy sets in X which are non-empty of order α (resp. order α*) and α-disjoint (resp. α*-disjoint).

Definition

Let $0 \leq \alpha < 1$ $(0 \leq \alpha < 1)$. A fuzzy topological space (X,T) is said to be **locally α-connect** (resp. locally α*-compact) if for each $p \in X$ there exists an open fuzzy set N such that $N(p) > \alpha$ (resp. $N(p) \geq \alpha$) α(N) (resp. α*(N)) is α-compact.

Definition

If G is any fuzzy set in a set X and $0 \le \alpha < 1$ then $(G) = \{x \in X : G(x) > \alpha\}$ (resp. $\alpha * (G) = \{x \in X : G(x) \ge \alpha\}$ is called an **α-level** (resp. α*-level) set in X.

Theorem

Let $0 \le \alpha < 1$. If a fuzzy topological space (X, τ) is α-Hausdorff, then is locally minimal.

Proof

Let $x, y \in X$ with x not equal to y.
There are G, H in T such that
$G(x) > \alpha,\ H(y) > \alpha$ and $G \wedge H = 0.$
To prove
(X, T_α) is Hausdorff space.
Then (G) and (H) are open sets in (X, T_α) and $X \in (G)$ and $Y \in \alpha(G).$
Also $(G) \cap \alpha(H) = \phi$
Since $G \wedge H = 0.$
Hence (X, T_α) is Hausdorff topological space

Conversely,

Suppose in the case of $\alpha = 0$
Let $x, y \in X$ with x not equal to y.
Then there are open sets U, V in (X, T_0) such that
$X \in U,\ Y \in V$ and $U \cap V = \phi$
Let U=0(G), V=0(H) for some G, H in T.
Then it follows that G(x)>0 and H(y)>0.
Further $G \wedge H = 0$ as $U \cap V = \phi.$
Hence (X,T) is 0-Hausdorff.
We have (X, T_α) is Hausdorff space.
Hence the proof.

ON α-N-TOPOLOGICAL SPACES ASSOCIATED WITH FUZZY TOPOLOGICAL SPACE

In this chapter, we introduced the concept of α-N-topological space associated with fuzzy topological space.

Definition

Let (X,T) be a topological space, $\alpha \in [0,1]$ and let $T = \{0,1\} \cup (U : U \in T\}$ then any $U \in T$ is a **α-neighbourhood** for any point $X \in U$ (i.e) For any point of its support, but obviously U fails to be α-open in (X,T).

Definition

Let (X,T) be a topological space, $\alpha \in [0,1]$ and let $T = \{0,1\} \cup \{\alpha.u : u \in T\}$ then any $u \in T$ is a **α-neighbourhood** for any point $x \in u$ (i.e) For any point of its support, but obviously u fails to be α-open in (X,T).

Definition

A fuzzy topological space (X,τ) is called **α-compact**if every-shading of X by fuzzy open sets of X has a finite α-subshading.

Definition

Let $0 \le \alpha < 1$. A fuzzy topological space (X,τ) is said to be **α-Hausdorff** if for each $x, y \in X$ with $x \neq y$ there exist fuzzy open sets A and B in X, such that $A(x) > \alpha, B(x) > \alpha$ and $A \wedge B = 0_x$.

Definition

Let (X,τ) be a fuzzy topological space. A subset A of X is said to be **α-N-open set** in X if its complement A^1 is **α-closed** in X.

Definition

Let (X,τ) be a fuzzy topological space. A subset A of X is said to be **α-N-open set** in X if its complement A^1 is **α-semiclosed**in X.

Definition

α-semiclosure of a set $A \subseteq X$, denote by **α-SclA**, is defined as α-Scl $A = A \cup A^{\alpha-slp}$.

Theorem

Let (X,T) be a fuzzy topological spaces and let $\alpha \in I$ for each point

i) If $M \in N_\alpha(p)$ then $p \in M$

ii) If $M, N \in N_\alpha$ then $M \cap N \in N_\alpha(p)$

iii) If $M \in N_\alpha(p)$ and $M \subset N$ then $N \in N_\alpha(p)$

iv) If $M \in N_\alpha(p)$ then there is $N \in N_\alpha(p)$ such that $\sigma(N) \ge \alpha$
$N \subset M$ and $N \in N_\alpha(q)$ for every $q \in N$.

Proof

i) If $M \in N_\alpha(p)$

To prove $p \in M$

Thrn there exist $G \in T_\alpha$

Such that $p \in G \subset M$ and therefore $p \in M$

Hence $p \in M$

ii) If $M, N \in N_\alpha(p)$

To prove $M \cap N \in N_\alpha(p)$

There exist two α-open fuzzy sets that is G_1 and G_2 such that

$$p \in G_1 \subset M, p \in G_2 \subset N$$

Since $G_1 \cap G_2 \in T_\alpha$ and $p \in G_1 \cap G_2 \subset M \cap N$

We have $M \cap N \in N_\alpha(p)$

iii) If $M \in N_\alpha(p)$

To prove $N \in N_\alpha(p)$

There exists $G \in T_\alpha$ such that $p \in G \subset M \subset N$ and

Hence $N \in N_\alpha(p)$

iv) Suppose $M \in N_\alpha(p)$

To prove $(N) \geq \alpha$ and $N \in N_\alpha(q)$ for every $q \in N$

Then there exists $G \in T_\alpha$ such that $p \in G \subset M$

Let N=G

We have $(N) \geq \alpha$ and $N \in N_\alpha(q)$ for every $q \in N$

Hence the proof.

Example

Let X be a set with atleast two points and $\alpha \in [0,1]$

Let {M,N} be a partition of X

we define the following fuzzy sets A and B

A(x)=α if $x \in M$ and A(x)=0 if $x \in N$

B(x)=α if $x \in M$ and B(x)=α/2 if $x \in N$

We have that A is an α-set and $A \subset B$ but B is not.

CONCLUSION

Thus we have given that in the proceeding chapter, we discussed on the level spaces of fuzzy topological spaces and also discussed on α-connected associated with fuzzy topological space to prove certain theorems using fuzzy topological space (X,T) is α-compact iff the corresponding α-level topological space (X,T) is compact and some important theorems are discuss these chapters with some valuable result using elaborated. The result on theorems and definitions a will be useful for research field.

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