ISOMORPHISM ON FUZZY GRAPHS

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Abstract

In this paper, the order, size and degree of the nodes of the isomorphic fuzzy graphs are discussed. Isomorphism between fuzzy graphs is proved to be an equivalence relation. Some properties of self-complementary and self-weak complementary fuzzy graphs are discussed. Keywords— complementary fuzzy graphs, co-weak isomorphism, equivalence relation, fuzzy relation, weak isomorphism.

1. INTRODUCTION

A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. So the graphs are simply models of relations.

When there is vagueness in the description of objects or in its relationships or in both, it is natural to assign a "**ISOMORPHISM ON FUZZY GRAPHS**".

Generally an undirected graph is a symmetric binary relation on a non- empty vertex set V. A fuzzy graph (undirected) is also a symmetric binary fuzzy relation on a fuzzy subset. Rosenfeld considered fuzzy relation on fuzzy sets and developed the theory of fuzzy graphs in 1975.

Although the first definition of fuzzy graphs was given by Kaufman,R.T.Yeh and S.V.Bang have also introduced various connectedness concepts in fuzzy graphs during the same time. The concept of isomorphism was also introduced by R.T.Yeh and S.V.Bang**.**

"Fuzzy set theory is a marvellous tool for modelling the kind of uncertainty associated with vagueness, with Imprecision and with a lack of information regarding to a particular element of the problem at hand"

Thus the idea of fuzziness is one of "enrichment" not of "replacement".

Zadeh's ideas have found application in computer science, artificial intelligence, decision analysis, information science, system science, control engineering, expert systems, pattern recognition, management science, operation research and robotics.

PRELIMINARIES

Fuzzy set theory was introduced by Zadeh in 1965.

Definition 1.1

The concept of a **fuzzy set** is an extension of the concept of a crisp set.

Just as a crisp set on a universal set U is defined by its characteristic function from U to {0,1}, a fuzzy set on a domain U is defined by its membership function from U to [0,1].

Let U be a non-empty set, to be called the universal set (or) the universe of discourse (or) simply a domain.

Then by a fuzzy set on U is meant a function, $A:U \to [0,1]$. 'A' is called the **membership function**, A(x) is called the **membership grade** of x.

We also write,

$$A = \{x, A(x) : x \in U\}$$

Example

Let U = {1, 2, 3, 4, 5}.

A = The set of all numbers in U very close to 1.

Where $A(x) = \frac{1}{x}$

⇒ A = {(1, 1), (2, 0.5), (3, 0.3), (4, 0.25), (5, 0.2)}

Definition 1.2

Let σ and τ be two fuzzy sets of a set V. Then σ is said to be **fuzzy subset** of τ, written as $\sigma \subseteq \tau$, if $\sigma(x) \le \tau(x)$ for every $u \in V$.

Definition 1.3

A is said to be **included (or) contained** in B, iff $A(x) \le B(x)$ for all x in U. In symbol we write

$A \subseteq B,$

$(i.e)$ A is a subset of B.

Definition 1.4

A is said to be **equal** to B (or) same as B, iff $A \subseteq B \,\&\, B \subseteq A.$

$(i.e),$ $A(x) = B(x)$ all in U.

we write $A = B.$

Definition 1.5

The **union** of A and B is denoted by $A \cup B$ and it is defined on U as,

$$A \cup B(x) = \max\{A(x), B(x)\}$$
$$= A(x) \vee B(x) \quad \text{for every x in U.}$$

Where '$\vee$' refers to the **maximum** of two fuzzy sets.

Properties of isomorphism on fuzzy graphs

i. A weak isomorphism preserves the weights of the nodes but not necessarily the weights of the edges.
ii. A co-weak isomorphism preserves the weight of the edges but not necessarily the weights of the nodes.
iii. An isomorphism preserves the weights of the edges and the weights of the nodes.
iv. An endomorphism of a fuzzy graph $G : (\sigma, \mu)$ is a homomorphism of G to itself.
v. An automorphism of a fuzzy graph $G : (\sigma, \mu)$ is an isomorphism of G to itself.
vi. When the two fuzzy graphs G and G' are same the weak isomorphism between them becomes an isomorphism and similarly the co-weak isomorphism between them also becomes isomorphism.
vii. In crisp graphs when two graphs are isomorphic they are of same size and order.

ISOMORPHIC GRAPHS AND THEIR COMPLEMENTS

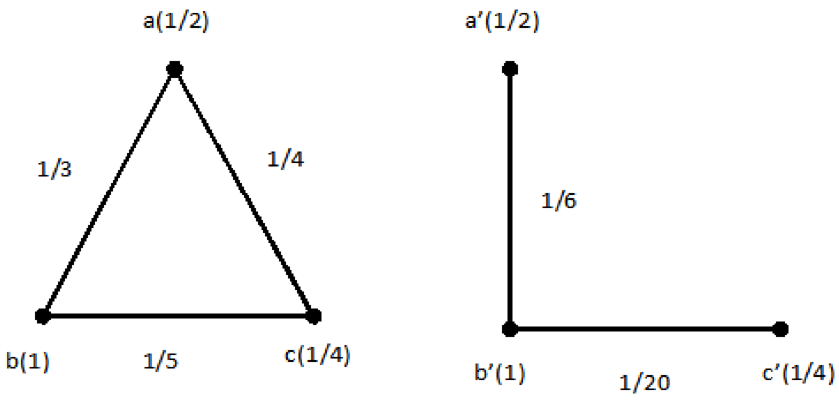
The following chapter discusses the results regarding isomorophic graphs and their complements and contains few results which relates isomorphically the fuzzy graph and its subgraph (or) its fuzzy line graph.

Definition 3.1

Let $G : (\sigma, \mu)$ be a fuzzy graph. The complement of G is defined as $\overline{G} : (\sigma, \overline{\mu})$

where

$$\overline{\mu}(x,y)=\sigma(x)\wedge\sigma(y)-\mu(x,y) \qquad \forall x,y\in S$$



$G:(\sigma,\mu)$ $\qquad\qquad$ $\overline{G}':(\sigma',\mu')$

Theorem 3.2

Two fuzzy graphs are isomorphic if and only if their complements are isomorphic.

Proof:

Let $G:(\sigma,\mu), G':(\sigma',\mu')$, be the two fuzzy graphs.

Assume $\overline{G}\cong\overline{G'}$

Claim:

$\overline{G}\cong\overline{G'}$

Since G is isomorphic to G' there exists a bijective map $h:S\to S'$ satisfying

$\sigma(x)=\sigma'(h(x)), \qquad \forall x\in S$ ---------(1)

$\mu(x,y)=\mu'(h(x),h(y)), \qquad \forall x,y\in S$ ---------(2)

From the definition of complementary fuzzy graphs,
we have

$\overline{\mu}(x,y)=(\sigma(x)\wedge\sigma(y))-\mu(x,y), \qquad \forall x,y\in S$

$\mu(x,y)=(\sigma'(h(x))\wedge\sigma'(h(y)))-\mu(x,y)$

and using equation 2

$\Rightarrow \mu(x,y)=(\sigma'(h(x))\wedge\sigma'(h(y)))-\mu'(h(x),h(y))$

$\therefore$ We have

$\Rightarrow \mu(x,y)=\mu(h(x),h(y))$ (by the definition of complement)

Hence we have a bijective map, $h:S\to S$ Satisfying

$\sigma(x)=\sigma'(h(x)), \qquad \forall x\in S$

And $\qquad \mu(x,y)\le\mu'(h(x),h(y)), \qquad \forall x,y\in S$

$\Rightarrow \overline{G}\cong\overline{G'}$

Conversely,

Assume that $\overline{G}\cong\overline{G'}$

Claim:

$\overline{G} \cong \overline{G'}$

Since $\overline{G} \cong \overline{G'}$, there exists a map $g : S \to S'$ which is a bijective map such that

$\sigma(x) = \sigma'(g(x)), \quad \forall x \in S$

And $\mu(x, y) \leq \mu'(g(x), g(y)), \quad \forall x, y \in S$ ----------- (3)

Using the definition of complement
we have,

$$\overline{\mu}(u, v) = \sigma(u) \wedge \sigma(v) - \mu(u, v)$$

$$\overline{\mu'}(g(x), g(y)) = \sigma'(g(x) \wedge \sigma'(g(y)) - \mu'(g(x), g(y))), \quad \forall x, y \in S$$

By using (3), we get

$$\mu(x, y) \leq \overline{\mu}(g(x), g(y)), \quad \forall x, y \in S$$

Hence we have a bijective map $g : S \to S'$ from G into $\bar{G}$ such that

$$\sigma(x) = \sigma'(g(x)), \quad \forall x \in S$$

And $\mu(x, y) \leq \mu'(g(x), g(y)), \quad \forall x, y \in S$

$$\Rightarrow \overline{G} \cong \overline{G'}$$

Hence the theorem.

ISOMORPHISM AND STRONG FUZZY GRAPHS:

Theorem: 5.4

If G is connected, strong fuzzy graph then every arc in G is a strong arc.

Proof:

G is a strong fuzzy graph, therefore $\mu(x, y) = \sigma(x) \wedge \sigma(y) \forall (x, y)$ in μ.

Case: 1

If (x, y) is the only arc connecting the nodes x and y then $\mu(x, y) = \mu^{\infty}(x, y)$.

Case: 2

If there are paths connecting the nodes x and y other than the edges (x, y) then consider an arbitrary path $\rho : x = u_0, u_1, u_2, u_3, \ldots\ldots\ldots, u_n = y$.

Strength of $\rho = \bigwedge_{i=1}^{n} \mu(u_{i-1}, u_i)$

$= \bigwedge_{i=1}^{n} \sigma(u_{i-1}), \sigma(u_i)$ since G is a strong fuzzy graph.

$= \min_{0 \leq i \leq n} \sigma(u_i) \leq \sigma(x) \wedge \sigma(y)$.

$\mu^{\infty}(x, y) =$ sup {strength of all paths connecting x and y }

$\leq \sigma(x) \wedge \sigma(y)$

$= \mu(x, y)$ since G is a strong fuzzy graph)

i.e. $\mu(x, y) = \mu^{\infty}(x, y)$.

Hence (x, y) is a strong arc in G by both cases.

CONCLUSION

In this paper isomorphism between fuzzy graphs is proved to be an equivalence relation and weak isomorphism is proved to be a partial order relation. Similarly it is expected that coweak isomorphism can be proved to be a partial order relation. A necessary and then a sufficient condition for a fuzzy graph to be self weak complementary are studied. The results discussed may be used to study about various fuzzy graph invariants.

In this thesis, it is proved the isomorphism between fuzzy graphs to be an equivalence relation and the weak isomorphism to be a partial order relation. When two fuzzy graphs are isomorphic/weak isomorphic/co- weak isomorphic, the relationship between their complements is dealt in detail. Self weak complementary. Also it is observed that the nature of the nodes are preserved under isomorphism, where as weak isomorphism preserves only busy nodes and co- weak isomorphism preserves only free nodes.

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