FUZZY USING FIBONACCI DIVISIOR CORDIAL GRAPHS

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ABSTRACT

Let G = (V, E) be a (p, q)-graph. A Fibonacci divisor cordial labeling of a graph G with vertex set V is a bijection f : V → {F1, F2, F3, . . . , Fp}, where Fi is the i th Fibonacci number such that if each edge uv is assigned the label 1 if f(u) divides f(v) or f(v) divides f(u) and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. If a graph has a Fibonacci divisor cordial labeling, then it is called Fibonacci divisor cordial graph. In this paper, we prove that the graphs Pn, Cn, K2,nu2(K1) and subdivision of bistar() are Fibonacci divisor cordial graphs. We also prove that Kn(n ≥ 3) is not Fibonacci divisor cordial graph. Keywords: Cordial labeling, divisor cordial labeling, Fibonacci divisor cordial labeling.

Key words: Fibonacci divisor cordial labeling of a graph G with vertex set V is a bijection f.

1. INTRODUCTION

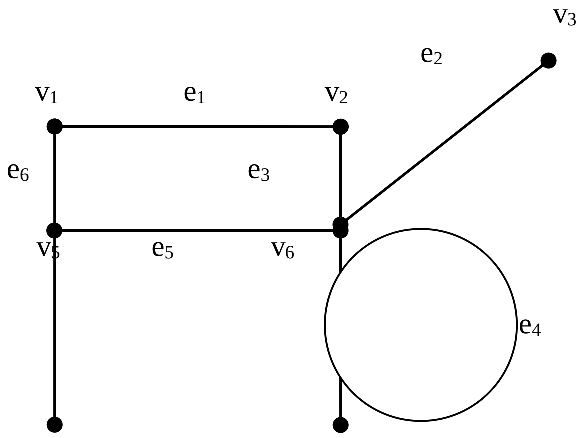
By a graph, we mean a finite, undirected graph without loops and multiple edges, for terms not defined here, we refer to Harary. In a labeling of a particular type, the vertices are assigned values from a given set, the edges have a prescribed induced labelings must satisfy certain properties. An excellent reference on this subject is the survey by Gallian. Two important types of labelings are called graceful and harmonious. Graceful labelings were introduced independently by Rosa in 1966 and Golombo in 1972, while harmonious labelings were first studied by Graham and Sloane in 1980. A third important type of labeling, which contains aspects of both of the other two is cordial and was introduced by Cahit in 1990. Whereas the label of an edge uv for graceful and harmonious labeling is given respectively by |f(u) − f(v)| and f(u) + f(v)(modulo q), cordial labeling use only labels 0 and 1 and the induced label f(u) + f(v)(modulo 2), which is of course equals |f(u) − f(v)|. Since arithmetic modulo 2 is an integral part of computer science, cordial labeling has close connections with that field. More precisely, a cordial graph is defined as follows.

2. PRELIMINARIES

DEFINITION

A graph G = (V,E) consists of a set of objects V = { v_1, v_2, v_3, . . . , v_n} of vertices and another set E = { e_1 , e_2 , e_3, ... e_n } of edges such that each edge e_k is identified with an unordered pair (v_i , v_j). Also A graph G with P vertices and Q edges is called a (P, Q) graphs.

EXAMPLE:



A (5,6) graph with V = { v_1, v_2, v_3, v_4, v_5} and E = { e_1, e_2 , e_3 , e_4, e_5, e_6}

DEFINITION:

An edge having the same vertex as both it's end vertices is called a **Self-loop.**

DEFINITION:

If more than one edge is associated with a given pair of vertices, the edges are called **parallel edges.**

DEFINITION:

Two vertices are said to be **adjacent vertices,** if they are incident with a common edge.

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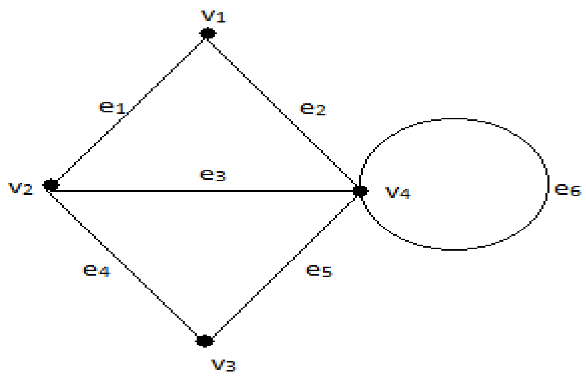
DEFINITION:

A graph with a finite number of vertices as well as a finite number of edges is called a **finite graph**.

EXAMPLE:



$d(v_1) = 2$ $d(v_2) = 3$ $d(v_3) = 2$ $d(v_4) = 5$

DEFINITION:

In a graph G = (V, E) it is possible for the edge set E to be empty. Such a graph without any edge is called a **null graph.**

DEFINITION:

A **walk** is a finite alternating sequence of vertices and edges beginning and ending with vertices such each edge is incident with vertices preceding and following it.

No edge can appear more than once, but vertices may be appearing more than once.

DEFINITION:

If the terminal vertices are distinct, a walk is called a **open walk.**

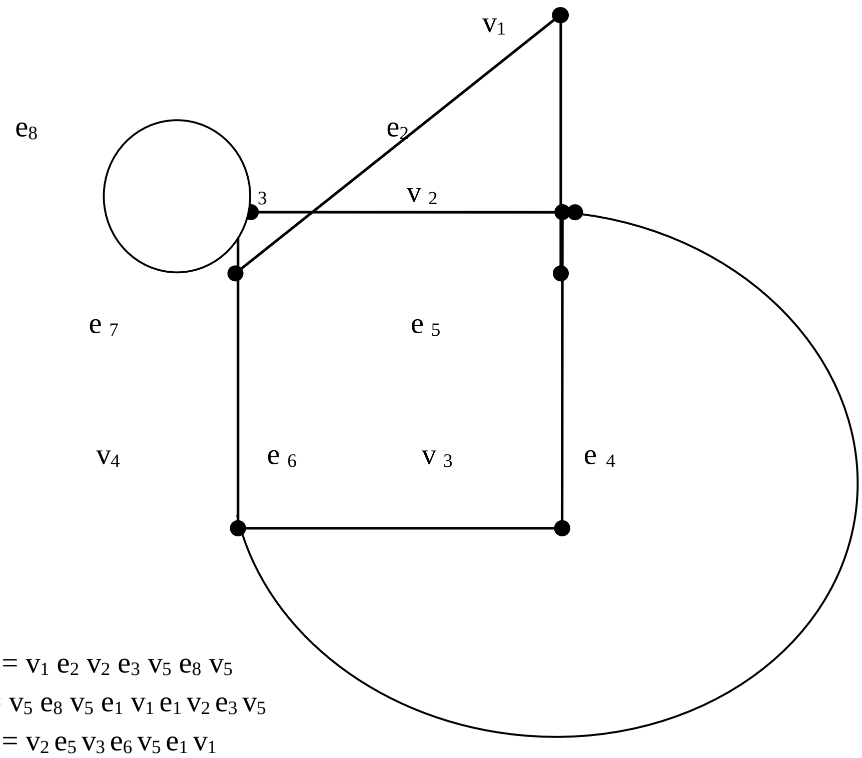
DEFINITION:

If the terminal vertices are the same, a walk is called **closed walk.**

DEFINITION:

An open walk in which, no vertex appears more than once is called **path**.

EXAMPLE:



Open walk $= v_1\ e_2\ v_2\ e_3\ v_5\ e_8\ v_5$

Closed walk $= v_5\ e_8\ v_5\ e_1\ v_1\ e_1\ v_2\ e_3\ v_5$

Path $= v_2\ e_5\ v_3\ e_6\ v_5\ e_1\ v_1$

DIVISOR CORDIAL GRAPHS

DEFINITION:

Let a and b be integers. If a divides b means that there is a positive integer k, such that b = ka. It is denoted by a / b.

DEFINITION:

The divisor function of integer d(n) is denoted by d(n) = ∑ 1. That is d(n) denotes the number of divisor of an integer n.

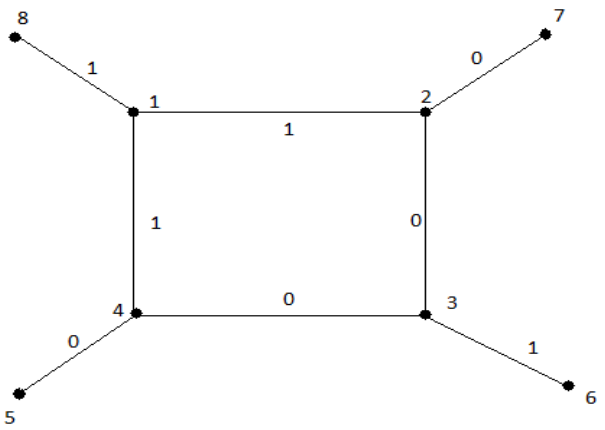
DEFINITION:

Let n be an integer and x be a real number. The divisor summability function is defined as $D(x) = \sum d(n)$. That is D(x) is the sum of the number of divisor of n for $n \leq x$.

DEFINITION:

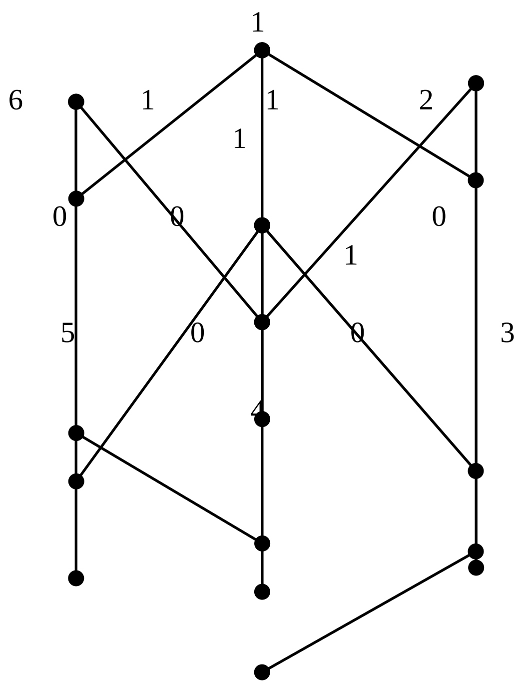
Let G = (V,E) be a simple graph and $f : v \rightarrow \{ 1,2, ... |v| \}$ be a bijection. For each edge uv, assign the label 1 if either $f(u) | f(v)$ or $f(v) | f(u)$ and the label 0 if $f(u) \nmid f(v)$ then f is called a **divisor cordial labelling** if $| e_f(0) - e_f(1) | \leq 1$. A graph with a divisor cordial labelling is called a **divisor cordial graph**.

EXAMPLE:



Consider the above graph G.
We have $e_f(0) = 4$ and $e_f(l)$
Then $| e_f(0) - e_f(1) | = | 4 - 4 | = 0$
Hence $| e_f(0) - e_f(1) | \leq 1$.
Hence G is divisor cordial graph.

EXAMPLE:



Consider the above graph G.
We have $e_f(0) = 5$ and $e_f(l) = 4$.
Then $|e_f(0) - e_f(1)| = |5 - 4| = 1$.
Therefore $|e_f(0) - e_f(1)| \leq 1$.
Hence G is divisor cordial graph.

THEOREM:

The cycle c_n is divisor cordial.

PROOF:

Let $v_1, v_2, \ldots\ldots, v_n$ be the vertices of the cycle c_n.

We follow that the same labelling pattern as in the path, except by interchanging the labels of v_1 and v_2.

Then if follows that

If n is odd, then $e_f(1) = \frac{n-1}{2}$ and $e_f(0)\ \frac{n-1}{2}$

If n is even, then $e_f(1) – e_f(0)\ \frac{n}{2}$

Hence $|e_f(0) - e_f(1)| \leq 1$

Therefore c_n is divisor cordial.

SPECIAL CLASSES OF DIVISOR CORDIAL GRAPH

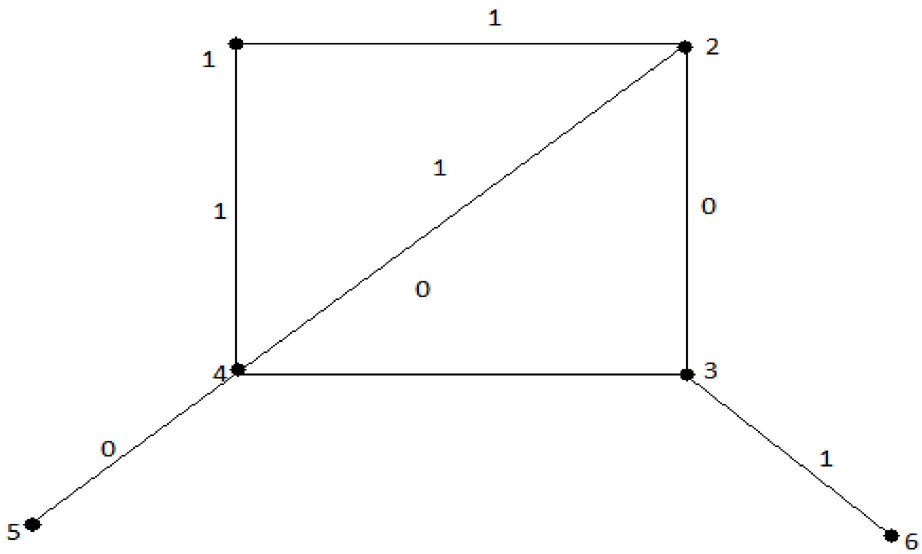
DEFINITION:

Let G = (V, E) be a simple graph and $f: V \to \{1, 2, \ldots |V|\}$ be a bijection. For each edge uv, assign the label 1 if either $f(u) \nmid f(v)$ or $f(v) \mid f(u)$ and the label 0 if $f(u) \mid f(v)$. Then f is called a divisor cordial labelling if $|e_f(0) – e_f(1)| \leq 1$.

A graph with a divisor cordial labelling is called a **divisor cordial graph.**

EXAMPLE:

Consider the following graph G



We have $e_f(0) = 3$ and $e_f(1) = 4$.

Then $|e_f(0) - e_f(1)| = ||3-4|| = 1$.

Hence $|e_f(0) - e_f(1)| \leq 1$

Hence G is divisor cordial.

THEOREM:

If G is a divisor cordial graph of even size, then G – e is also divisor cordial for all e ϵ E (G).

PROOF:

Let q be the even size of the divisor cordial graph G.

Then it follows that $e_f(0) = e_f(1) = \frac{q}{2}$

Let e be any edge in G which is labelled either 0 or 1.

Then in G- e, we have either

$e_f(0) = e_f(1) + 1$ or $e_f(1) = e_f(0) + 1$

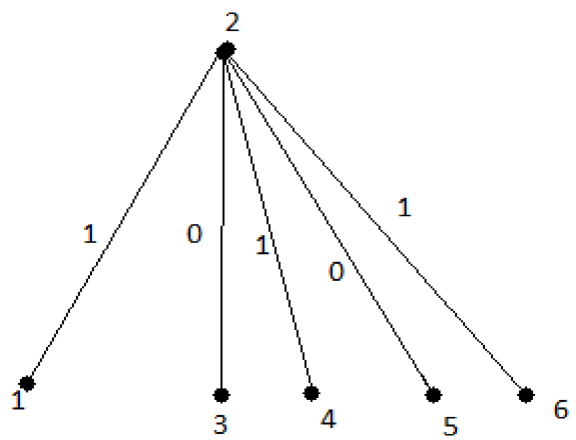
Hence $| e_f(0) - e_f(1) | \leq 1$.

Therefore G – e is divisor cordial graph.

EXAMPLE:

For even size G

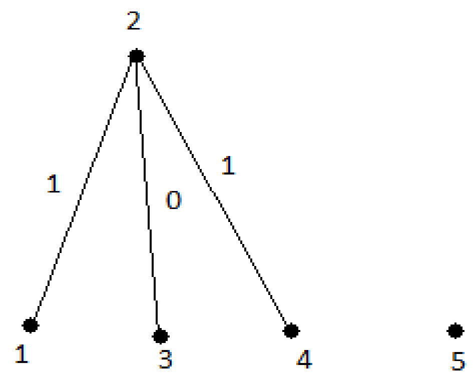
G:



We have $e_f(0) = e_f(1) = 2$

Then $| e_f(0) – e_f(1) | = | 2\text{-}2 | = 0$.

Hence $| e_f(0) - e_f(1) | \leq 1$



We have $e_f(0) = 1$ and $e_f(1) = 2$

Then $| e_f(0) - e_f(1) | = | 1-2 | = 1$.

Hence $| e_f(0) - e_f(1) | \leq 1$

Therefore G – e is divisor cordial graph.

THEOREM: 4.2

If G is divisor cordial graph of odd size, then G – e is also divisor cordial for some e ϵ E(G).

PROOF:

Let q be the odd size of the divisor cordial graph G.

Then it follows that either

$e_f(0) = e_f(1) + 1$ or $e_f(1) = e_f(0) + 1$

If $e_f(0) = e_f(1) + 1$ then remove the edge e which is labelled as 0.

If $e_f(1) = e_f(0) + 1$ then remove the edge e which is labelled 1 from G.

Then it follows that $e_f(0) = e_f(1)$.

Therefore G – e is divisor cordial for some e ϵ E(G).

THEOREM:

Let G be any divisor cordial graph of size m and $K_{2,n}$ be a bipartite graph with the bipartition V = V1 U V2 with $V_1 = \{x_1, x_2\}$ and $V_2 = \{y_1, y_2, . . . y_n\}$. Then the graph $G * K_{2,n}$ obtained by identifying the vertices x_1 and x_2 of $K_{2,n}$ with that labled 1 and the largest prime number p such that $p \leq m$ respectively in G is also divisor cordial.

PROOF:

Let G be any divisor cordial graph of size m.

Let v_k and v_l be the vertices having the lables 1 and the largest prime number namely p such that $p \leq m$.

Let V = V1 U V2 be the bipartition of $K_{2,n}$ such that $V_1 = \{x_1, x_2\}$ and $V_2 = \{y_1, y_2, . . . y_n\}$.

Now assign the lables m+1, m+2,....m+ n to the vertices $y_1, y_2, ...y_n$ respectively.

Now identify the vertices x_1 and x_2 of $K_{2,n}$ with that labeled 1and the largest p such that $p \leq m$ respectively in G.

Case (i) $p < m + n > 2p$

Then the multiples of p are not available in the labels of y_1 $(1 \leq i \leq n)$.

The edges of $K_{2,n}$ incident with the vertex v_k have the label 1 and with the vertex v_1 have the label 0.

Thus the edges of $K_{2,n}$ contribute equal numbers namely n to both $e_f(1)$ and $e_f(0)$ in $G * K_{2,n}$.

Hence $G * K_{2,n}$ is divisor cordial.

Case (ii) $m + n \geq 2p$

Let q be the largest prime number such that $q \leq m + n$ which is labeled to some y_i. Then interchange the labels of v_1 and y_1, that is p and q.

We note that the largest prime number q does not divide the labels of $y_1, y_2, \ldots, y_n$. So, again the edges of $K_{2,n}$ incident with the vertex v_1 have the labels 0 and hence $G*K_{2,n}$ is divisor cordial.

CONCLUSION

We introduced a new concept of Fibonacci Divisor Cordial Graph. In this dissertation we have seen introduction and some definitions of Graph theory.

Here we presented some definitions, theorems, and examples based on Fibonacci Divisor Cordial Graphs.

Here we have investigated nine new graph families which admit Fibonacci cordial labeling. Further we have discussed Fibonacci cordial labeling in context of vertex switching and joint sum of different graph families and derived fivemore results.

We explained that how the labels are presented in Graph theory. Here we proved some theorems based on Divisor Cordial Labelling.

Divisor Cordial Labelling is one of the most involved areas in graph theory.

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