Scope International Journal of Science, Humanities, Management and Technology. ISSN : 2455-068X
Vol.4 Issue 3 (2018) 119 - 127. Submitted 23/07/2018. Published 31/08/2018

TRIPLE CONNECTED DOMINATION NUMBER OF A GRAPH

Mr.N.Rajkumari *& V.Murugan**

*Assistant Professor, Department of Mathematics, PRIST University, Thanjavur, Tamil Nadu.

**Research Scholar, Department of Mathematics, PRIST University Thanjavur, Tamil Nadu.

Abstract

The concept of triple connected graphs with real life application was introduced in [7] by considering the existence of a path containing any three vertices of a graph G. In this paper, we introduce a new domination parameter, called Smarandachely triple connected domination number of a graph. A subset S of V of a nontrivial graph G is said to be Smarandachely triple connected dominating set, if S is a dominating set and the induced sub graph hSi is triple connected. The minimum cardinality taken over all Smarandachely triple connected dominating sets is called the Smarandachely triple connected domination number and is denoted by γtc. We determine this number for some standard graphs and obtain bounds for general graphs. Its relationship with other graph theoretical parameters are also investigated.

Key Words: Domination number, triple connected graph, Smarandachely triple connected domination number.

INTRODUCTION

One of the fastest growing areas in graph theory is the study of domination. It takes back to 1850's with the study of the problem of determining the minimum number of queen which are necessary to cover an n*n chessboard. More than 50 types of domination parameters have been studied by different authors. Ore, Berg introduced the concept of domination sets. Extensive research activity is going on in Domination set of graphs. Acharya B.D, SampathKumar.E, V.R Kulli, Waliker H.B are some of the Indian Mathematicians who have made substantial contribution to the study of domination in graphs.
Domination is applied in many fields. Some of them are.

1. Communication network
2. Facility location problem
3. Land surveying
4. Routings etc.,

BASIC DEFINITIONS

Graph

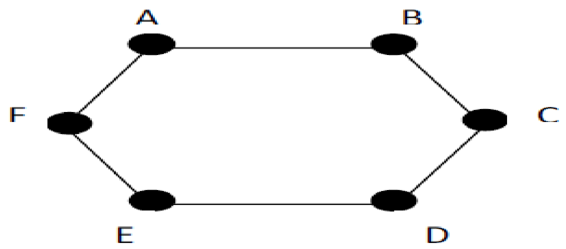
A graph consists of a set $V = \{v_1, v_2, \ldots, v_n\}$ called vertices and another set $E = \{e_1, e_2, \ldots, e_m\}$ whose element are called edges such that each edge e_k is identified with an unordered pair (v_i, v_j) of vertices, the vertices (v_i, v_j) associated with of the edge ek are called the end vertices of the edge e_k.

Order and Size of a graph

The number of vertices in V(G) is called the order of G and the number of edges in E(G) is called the size of G.

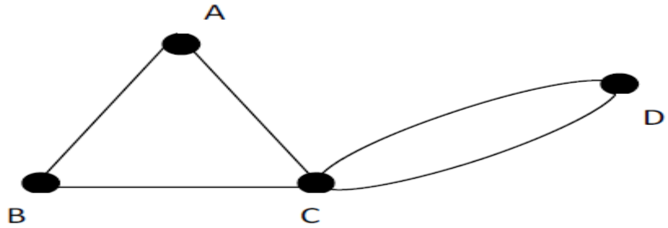
Simple Graph

A graph which has no loops and multiple edges is called a simple graph.



Multigraph

A graph which has multiple edges but no loops is called a multigraph.



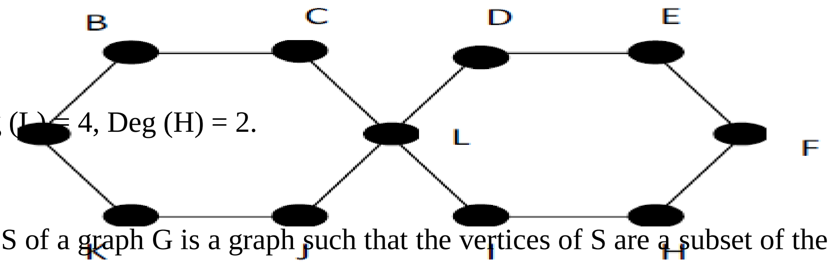
General graph

A graph which contains multiple edges or loops (or both) is called a general graph.

Degree of vertex

Let G is the graph with loops, and let v be a vertex of G. The degree of v is the number of edges meeting at v, and is denoted by deg (v).

The minimum degree of vertices of G is denoted by δ(G) and the maximum degree of vertices of G is denoted by Δ(G).

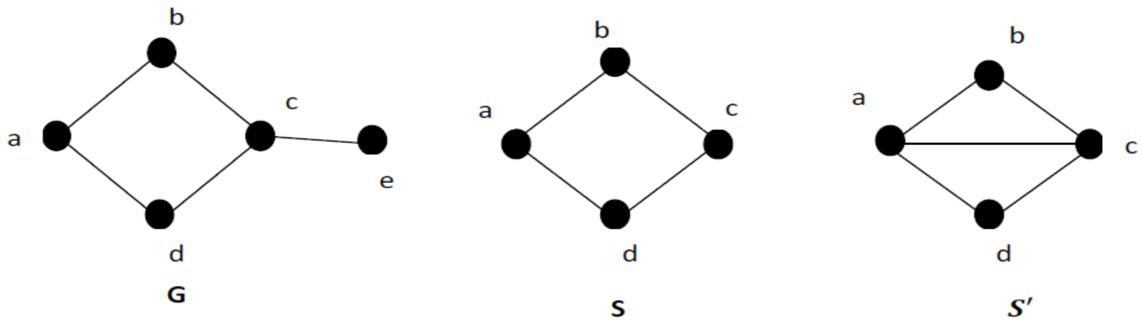


Deg (A) = 2, Deg (L) = 4, Deg (H) = 2.

(G) = 2, & Δ(G) = 4.

Subgraph

A subgraph S of a graph G is a graph such that the vertices of S are a subset of the vertices of G. (i.e.) V(S) ⊆ V(G) the edges of S are a subset of the edges of G.(i.e.)E(S) ⊆E(G).



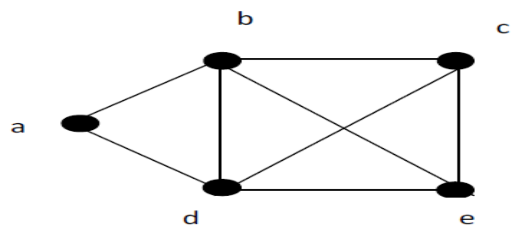
S is a subgraph of G

S^1 is not a subgraph of G

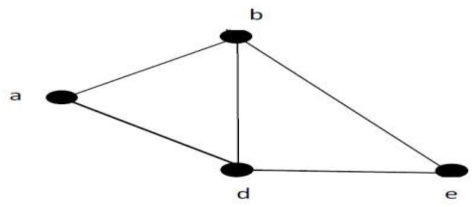
Induced Subgraph

A vertex-induced subgraph is one that consists of some of the vertices of the original graph and all of the edges that connect them in the original denoted by ⟨V⟩.

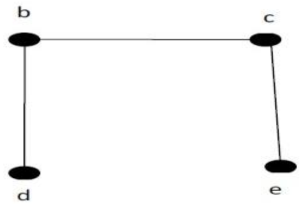
Example :



G



G_1



G_2

G_1 is an induced subgraph - induced by the set of vertices $V_1 = \{a, b, d, e\}$.
G_2 is not an induced subgraph.

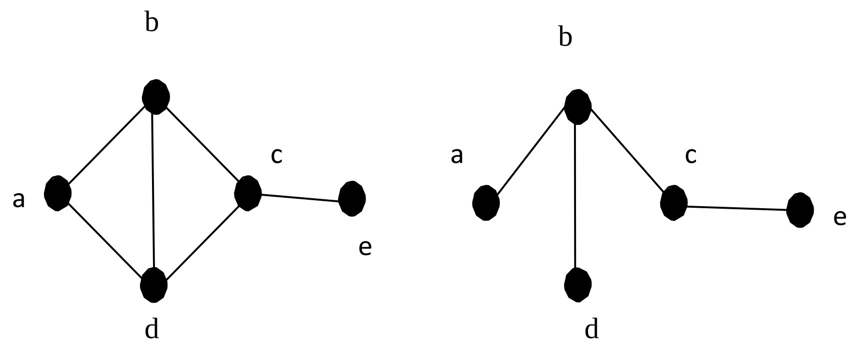
1.9 Proper subgraph

If S is a subgraph of G then we write $S \subseteq G$. When $S \subseteq G$ but $S \neq G$.

Spanning subgraph

A spanning subgraph of G is a subgraph that contains all the vertices of G. (i.e.) V(S) = V(G). S is spanning subgraph of G.

Example :



TRIPLE CONNECTED DOMINATION NUMBER OF A GRAPH

2.1Definition

A dominating set S of a connected graph G is said to be a triple connected dominating set of G if the induced sub graph < S > is triple connected.

The minimum cardinality taken over all triple connected dominating sets is the triple connected domination number and is denoted by (G).

2.2 Theorem

A tree T is triple connected if and only if $T \cong P_p; P \geq 3$.

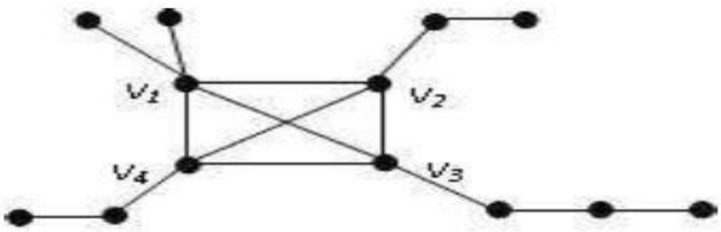
2.3 Theorem

A connected graph G is not triple connected if and only if there exists a H -cut with $\omega(G - H) \geq 3$ such that $|V(H) \cap N(C_i)| = 1$ for at least three components C_1, C_2 and C_3 of $G - H$.

Let G be a connected graph with m vertices $v_1, v_2, \ldots, v_m$. The graph obtained from G by attaching n_1 times a pendant vertex of P_{l_1} on the vertex v_1, n_2 times a pendant vertex of P_{l_2} on the vertex v_2 and so on, is denoted by $G(n_1P_{l_1}, n_2P_{l_2}, n_3P_{l_3}, \ldots, n_mP_{l_m})$ where $n_i, l_i \geq 0$ and $1 \leq i \leq m$.

Example :

Let v_1, v_2, v_3, v_4, be the vertices of K_4. The graph $K_4(2P_2, P_3, P_4, P_3)$ is obtained from K_4 by attaching 2 times a pendant vertex of P_2 on v_1, 1 time a pendant vertex of P_3 on v_2, 1 time a pendant vertex of P_4 on v_3 and 1 time a pendant vertex of P_3 on v_4.

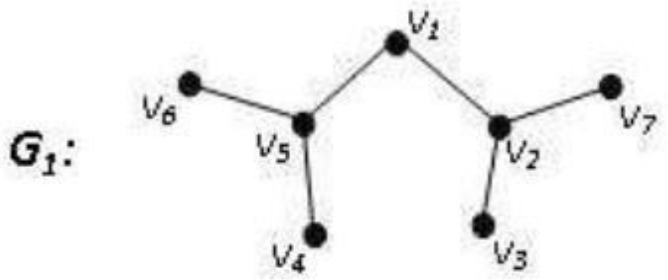


2.1 Definition

A subset S of V of a nontrivial connected graph G is said to be a Smarandachely triple connected dominating set, if S is a dominating set and the induced subgraph <S> is triple connected. The minimum cardinality taken over all Smarandachely triple connected dominating sets is called the Smarandachely triple connected domination number of G and is denoted by $\gamma_{tc}(G)$. Any Smarandachely triple connected dominating set with γ_{tc} vertices is called a γ_{tc}-set of G.

Example :

For the graph G_1 , S = {v_1, v_2, v_5} forms a γ_{tc}-set of G_1. Hence $\gamma_{tc}(G_1) = 3$.



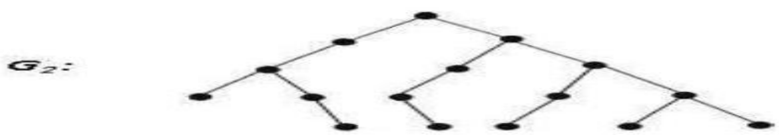
Graph with $\gamma_{tc} = 3$

Observation

Triple connected dominating set (tcd-set) does not exist for all graphs and if exists, then $\gamma_{tc}(G) \geq 3$.

Example :

For the graph G_2, any minimum dominating set must contain all the supports and any connected subgraph containing these supports is not triple connected and hence γ_{tc} does not exist.



PAIRED TRIPLE CONNECTED DOMINATION NUMBER OF A GRAPH

3.1 Definition

A subset S of V of a nontrivial graph G is said to be a paired triple connected dominating set, if S is a triple connected dominating set and the induced subgraph < S > has a perfect matching. The minimum cardinality taken over all paired triple connected dominating sets is called the paired triple connected domination number and is denoted by γ_{ptc}. Any paired triple connected dominating set with γ_{ptc} vertices is called a γ_{ptc}-set of G.

Example :

For the graph $C_5 = v_1v_2v_3v_4v_5v_1$, $S = \{v_1, v_2, v_3, v_4\}$ forms a paired triple connected dominating set. Hence $\gamma_{ptc}(C_5) = 4$.

3.2 Theorem

G is semi-complete graph with $p \geq 4$ vertices. Then G has a vertex of degree 2 if and only if one of the vertices of G has consequent neighborhood number $p - 3$.

3.3 Theorem

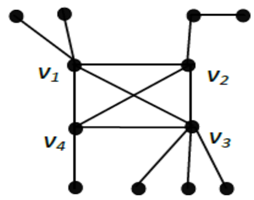
G is semi-complete graph with $p \geq 4$ vertices such that there is a vertex with consequent neighbourhood number $p - 3$. Then $\gamma(G) \leq 2$.

Let G be a connected graph with m vertices $v_1, v_2, v_2, \ldots \ldots, v_m$. The graph $G(n_1P_{l_1}, n_2P_{l_2}, n_3P_{l_3}, \ldots \ldots, n_mP_{l_m})$, where $n_i, l_i \geq 0$ and $0 \leq i \leq m$, is obtained from G by pasting n_1 times a pendant vertex of P_{l_1} on the vertex v_1, n_2 times a pendant vertex of P_{l_2} on the vertex v_1 and so on.

Example :

Let $v_1, v_2, v_3, v_4,$ be the vertices of K_4, the graph $K_4(2P_2, P_3, 3P_2, P_2)$ is obtained from K_4 by pasting 2 times a pendant vertex of P_2 on v_1, 1 times a pendant vertex of P_3on v_2, 3 times a pendant vertex of P_2 on v_3 and 1 times a pendant vertex of P_2 on v_4 and the graph in G_1.

G_1:



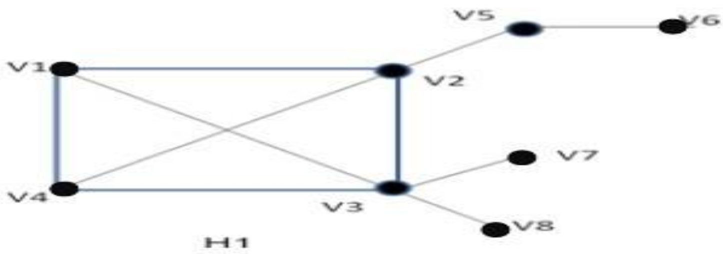
STRONG TRIPLE CONNECTED DOMINATION NUMBER OF A GRAPH

4.1Definition

A subset S of V of a nontrivial graph G is said to be a strong triple connected dominating set, if S is a strong dominating set and the induced subgraph <S> is triple connected. The minimum cardinality taken over all strong triple connected dominating sets is called the strong triple connected domination number of G and is denoted by γ_{stc} (G). Any strong triple connected dominating set with γ_{stc} vertices is called a γ_{stc} -set of G.

Example :

For the graph H1, S = {v1, v2, v3} forms a γ_{stc} -set of



Graphwithγ $_{\gamma stc = 3}$

4.2Theorem

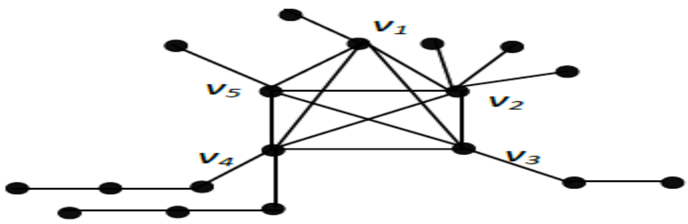
Let G be any graph and D be any dominating set of G. then $|V - D| \leq \sum_{u \in V(D)} \deg(u)$ and equality hold in this relation if and only if D has the following properties.

i. D is independent

ii. For every $u \in V - D$, there exists a unique vertex $v \in D$ such that $N(u) \cap D = \{v\}$.

Example :

Let v1, v2, v3, v4, be the vertices of K5. The graph K5(P2, 3P2, P3, 2P4, P2) is obtained from K5 by attaching 1 time a pendant vertex of P2 on v1, 3 time a pendant vertex of P2 on v2, 1 time a pendant vertex of P3 on v3 and 2 times a pendant vertex of P4 on v4, 1 time a pendant vertex of P2.



K5(P2, 3P2, P3, 2P4, P2)

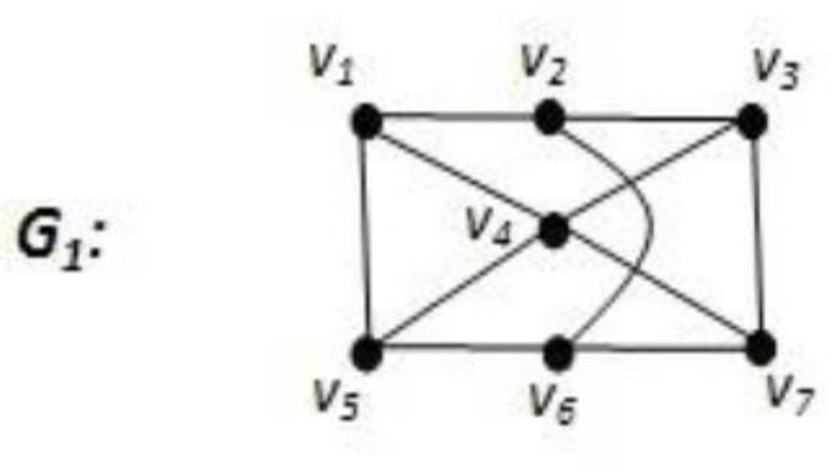
WEAK TRIPLE CONNECTED DOMINATION NUMBER OF A GRAPH

5.1 Definition

A subset S of V of a nontrivial graph G is said to be a weak triple connected dominating set, if S is a weak dominating set and the induced subgraph < S > is triple connected. The minimum cardinality taken over all weak triple connected dominating sets is called the weak triple connected domination number of G and is denoted by γ_{wtc} (G). Any weak triple connected dominating set with γ_{wtc} vertices is called a γ_{wtc} -set of G.

Example :

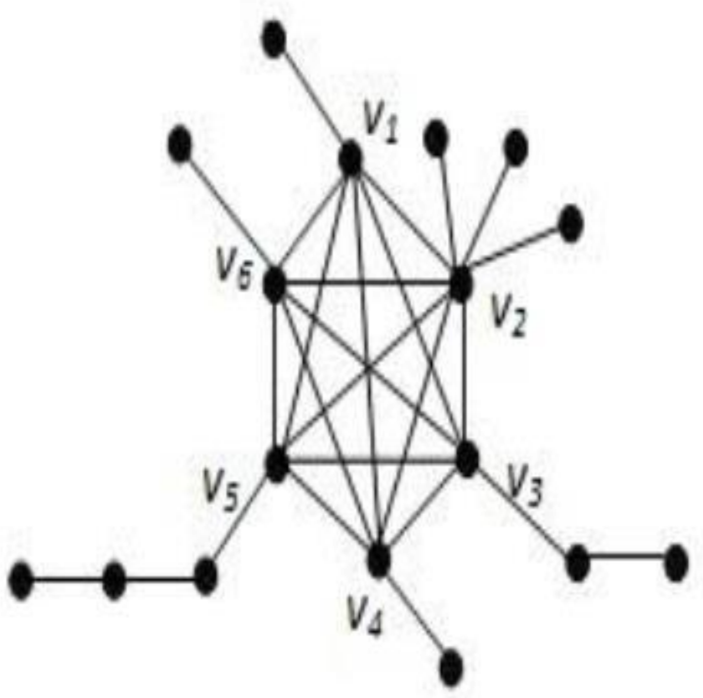
For the graph G_1, S = {v_1, v_2, v_3} forms a γ_{wtc} -set of G. Hence $\gamma_{wtc}(G_1)$ = 3.



Let G be a connected graph with m vertices v_1, v_2, …., v_m. The graph obtained from G by attaching n_1 times a pendant vertex of P_{l1} on the vertex v_1, n_2 times a pendant vertex of P_{l2} on the vertex v_2 and so on, is denoted by
$G(n_1P_{l1}, n_2P_{l2}, n_3P_{l3}, \ldots, n_mP_{lm})$ where $n_i, l_i \geq 0$ and $1 \leq i \leq m$.

Example :

Let v_1, v_2, v_3, v_4, v_5, v_6 be the vertices of K_6. The graph $K_6(P_2, 3P_2, P_3, P_2, P_4, P_2)$ is obtained from K_6 by attaching 1 time a pendant vertex of P_2 on v_1, 3 time a pendant vertex of P_2 on v_2, 1 time a pendant vertex of P_3 on v_3 and 1time a pendant vertex of P_2 on v_4, 1 time a pendant vertex of P_4 on v_5, 1 time a pendant vertex of P_2 on v_6 .



Observation

Weak triple connected dominating set (wtcd set) does not exists for all graphs and if exists, then $\gamma_{wtc}(G) \geq 3$.

Example :

For the graph G_2, any minimum triple connected dominating set must contain the v_5 and any triple connected dominating set containing v_5 is not a weak triple connected and hence γ_{wtc} does not exists.

Observation

The complement of the weak triple connected dominating set need not be a weak triple connected dominating set.

Observation

Every weak triple connected dominating set is a triple dominating set but not conversely.

Observation

Every weak triple connected dominating set is a dominating set but not conversely.

CONCLUSION

The concept of triple connected digraphs and domination in triple connected digraphs can be applied to physical problems such as flow networks with valves in the pipes and electrical networks, neural networks etc. They are applied in abstract representations of computer programs and are an invaluable tools in the study of sequential machines. In future this paper can be extended to studies of strong and weak domination in triple connected digraphs.

REFERENCES

1. Mahadevan G., Selvam A., Paulraj Joseph J., Ayisha B., and Subramanian T,Complementary triple connected domination number of a graph , Accepted for publication in Advances and Applications in Discrete Mathematics, ISSN 0974-1658(2012).
2. Mahadevan. G, Selvam Avadayappan, Nagarajan. A, Rajeswari. A, Subramanian. T, Paired Triple connected domination number of a graph, International Journal of Computational Engineering Research, Vol. 2, Issue 5, Sep. 2012, pp. 1333-1338(2012).
3. Mahadevan G., Selvam Avadayappan., Hajmeeral . M., and Subramanian T, Dom strong triple connected domination number of a graph, American Journal of Mathematics and Mathematical Sciences, ISSN 2278-0874, Vol. 1, Issue No. 2, July – dec. 2012, pp.29-37.
4. Mahadevan G., Selvam A., Bhagavathi Ammal V. G, and Subramanian T, weak triple connected domination number of a graph, International Journal of Modern Engineering Research (IJMER) Vol.3, Issue.1, Jan-Feb. 2013 pp-342-345 ISSN: 2249-6645.
5. Mahadevan G., Selvam A., Bhagavathi Ammal V. G, and Subramanian T, Strong triple connected domination number of a graph, International Journal Of Computational Engineering Research (ijceronline.com) Vol. 3 Issue. 1 (2012).
6. Nordhaus E. A. and Gaddum J. W. On complementary graphs, Amer. Math. Monthly, 63: 175–177 (1956).
7. Paulraj Joseph J., Angel Jebitha M.K., Chithra Devi P. and Sudhana G. Triple connected graphs, Indian Journal of Mathematics and Mathematical Sciences, ISSN 0973-3329,Vol. 8, No.1, pp 61-75 (2012).
8. Sampathkumar, E.; Walikar, HB The connected domination number of a graph, J.Math. Phys. Sci 13 (6): 607–613 (1979).
9. Teresa W. Haynes, Stephen T. Hedetniemi and Peter J. Slater, Fundamentals of domination in graphs, Marcel Dekker, New York (1998).