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FUZZY TRANSPORTATION PROBLEM WITH FUZZY DEMAND AND SUPPLY

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ABSTRACT

In this thesis, we shall study the fuzzy transportation problem with fuzzy quantities, and also we define the fuzzy triangular numbers and fuzzy transportation costs. To solve of optimum solution FTP used fuzzy transportation algorithm with least cost method. In this project entitled "Fuzzy Transportation Problem" is the consolidation of different notions that enrich the realm of the transportation problems in fuzzy environment. More particularly, the project contemplates different transportation algorithms pertaining to the fuzzy transportation problems. Better solution procedures have been provided for solving fuzzy transportation problems in the fuzzy environment yielding from the traditional way of solving.

Keywords: Fuzzy set, Fuzzy transportation problem, Trapezoidal Fuzzy number, Ranking Technique.

INTRODUCTION

This introductory chapter discusses the transportation problems, fundamental concepts of fuzzy set theory, membership functions, fuzzy numbers and defuzzification methods. A brief note on fuzzy transportation problem is also mentioned.

The basic transportation problem was originally developed by Hitchcock. The transportation problems can be modelled as a standard linear programming problem. In a typical product is to be transported from m sources to n designations and their capacities are a1, a2,.. , am and b1,b2,...,bn respectively. In addition there is a penalty cij associated with transporting unit of product from source i to destination j. The transportation problem is one of the earliest applications of linear programming problems. The objective function is to minimize total transportation costs and satisfy the destination requirements within the source availability. In order to solve a transportation the decision parameters of the problem must be fixed at crisp values.

Transportation problems have been widely studied in Computer Science and Operations Research. It is one of the fundamental problems of network flow problem which is usually use to minimize the transportation cost for industries with number of sources and number of destination while satisfying the supply limit and demand requirement. Transportation models play an important role in logistics and supply-chain management for reducing cost and improving service. Some previous studies have devised solution procedure for the transportation problem with precise supply and demand parameters. Efficient algorithms have been developed for solving the transportation problem when the cost coefficients and the supply and demand quantities are known exactly. In real world applications, the supply and demand quantities in the transportation problem are sometimes hardly specified precisely because of changing economic conditions. It was first studied by F. L. Hitchcock in 1941, then separately by T. C. Koopmans in 1947, and finally placed in the framework of linear programming and solved by simplex method by G. B. Dantzing in 1951.

The basic steps to solve transportation problems are:

Step 1. Determination of Initial Basic Feasible Solution (IBFS)

Step 2. Determination of optimal solution using the IBFS

Several heuristic methods are available to get an initial basic feasible solution. Although some heuristics can find IBFS very quickly and its solution are they find is often very good in terms of minimizing total cost. Well known heuristics methods are North-West corner rule, Row minima method, Column minima method, Matrix Minima Method, Vogel's Approximation Method.

The Idea of Fuzzy Set was introduced by Zadeh in 1965.Bellmanand and Zadeh proposed the concept of decision making in fuzzy environment. Fuzzy set theory is an excellent mathematical tool to handle the uncertainty arising the due to vagueness. The theory proposes a mathematical technique for dealing with imprecise concepts and problems that have many possible solutions.

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Fuzzy transportation problem (FTP) is the problem of minimizing fuzzy valued objective functions with fuzzy source and fuzzy destination parameters. The balanced is both a necessary and sufficient condition for the existence of a feasible solution to the transportation problem. Shan Huo Chen introduced the concept of function principle that is used to calculate the fuzzy transportation cost. The Graded Mean Integration Representation Method, used to defuzzify the fuzzy transportation cost, was also introduced by Shan Huo Chen.

In a fuzzy transportation problem, all parameters are fuzzy numbers. Fuzzy numbers may be normal or abnormal, triangular or trapezoidal. Ranking method is used to change the fuzzy number into crisp form. The method for ranking was first proposed by Jain. The aim of fuzzy transportation is to find the least transportation cost of some commodities through a capacitated network when the supply and demand of nodes and the capacity and cost of edges are represented. Their method is determined efficient solutions for the transformed problem; nevertheless, only crisp solutions were provided. If the cost coefficients or the demand and supply quantities are fuzzy numbers, the total transportation cost will be fuzzy as well.

BASIC DEFINITIONS AND PRELIMINARIES

DEFINITION 2.1

A set is a well defined collection of objects.

2.1.1 Example The collection of all natural numbers (i.e) N= {1, 2, 3, 4…}

DEFINTION 2.2

A set defined using a characteristic function that assign a value of either 0 or 1 to each element of the universe thereby discriminating between members and non-members of the crisp set under consideration.

DEFINITION 2.3

Let x be a non-empty set. A fuzzy set A in X is characterized by its membership function. $A=\{(x,\mu_A(x)):X\in A,\mu_A(x)\in[0,1]\}$ and $\mu_A(x)$ is interpreted as the degree of membership of element x in fuzzy set A for each x∈X.

2.3.1 Example: A = {(1,.4),(2,.5),(3,.6),(4,.8)}

DEFINITION 2.4

A convex and normalized fuzzy set defined on R whose membership function is piecewise continuous is called Fuzzy number.

DEFINITION 2.5

A Fuzzy number A is a fuzzy set of the real line with a normal, (fuzzy) convex and continuous membership function of bounded support. The family of fuzzy numbers will be denoted by F

DEFINITION 2.6

A fuzzy number A in R said to be a Triangular fuzzy number if its membership function $\mu_A(x)$ has the following form:

$$\mu_A(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1} & if \quad a_1\le x\le a_2\\ \dfrac{x-a_3}{a_2-a_3} & if \quad a_2\le x\le a_3\\ 0 & otherwise\end{cases}$$

FUZZY TRANSPORTATION PROBLEM WITH FUZZY DEMAND AND SUPPLY

PROBLEM 3.1

A company has three factories and five warehouses. The commodities are transported from the factories to the warehouses, which are located at various distances from the factories. The warehouse requirements are d_1= (10,20,30), d_2=(15,20,25), d_3=(15,25,35), d_4=(10,30,50) ,d_5=(10,25,40). The requirements of each factory are given by, s_1=(25,30,35), s_2=(25,40,55), s_3=(40,50,60). The table below shows the costs of transportation from factories to warehouses.

						SUPPLY
	7	10	14	8	0	(25,30,35)
	7	12	12	6	0	(25,40,55)
	5	8	15	9	0	(40,50,60)
DEMAND	(10,20,30)	(15,20,25)	(15,25,35)	(10,30,50)	(10,25,40)	

3.1.1 NORTH WEST CORNER RULE METHOD

ITERATION 1

Select the north-west cell for allocation. In the table below, the minimum entry is "(7)" in cell (1,1). Allocation for cell (1,1) = min(10,20,30),(25,30,35) = (10,20,30). Allocate (10,20,30) units to cell (1,4).

						SUPPLY
	(10,20,30) 7	10	14	8	0	(25,30,35)
	7	12	12	6	0	(25,40,55)
	5	8	15	9	0	(40,50,60)
DEMAND	(10,20,30)	(15,20,25)	(15,25,35)	(10,30,50)	(10,25,40)	

Proceeding in this way we get the final iteration,

FINAL ITERATION

Check whether all the cells are allocated. If not, repeat the process. Thus the fuzzy initial basic feasible solution is:

(10, 20, 30) 7	(-5, 10, 25) 10	14	8	0
7	(-10, 10, 30) 12	(15,25,35) 12	(-40, 5, 50) 6	0
5	8	15	(-40, 25, 90) 9	(-50, 25, 100) 0

Fuzzy transportation cost

Z = (10,20,30)×7 + (-5,10,25)× 10 + (15,25,35)×12 + (-40,5,50)× 6 + (-10,10,30) ×12+ (-40, 25, 90) ×9 + (-50, 25,100) × 0

= (70,140,210) + (-50,100,250) + (-120,120,360) + (180,300,420) + (-240,30,300) + (-360,225,810) + (0,0,0)

= (-520,915,2350)

Z = 915

Defuzzified value is 915

3.1.2 ROW MINIMA METHOD

ITERATION 1

Select the least expensive in a first row cell for allocation. In the table below, the minimum entry is "(0)" in cell (1,5). Allocation for cell (1,5) = min(10,25,40),(25,30,35) = (10,25,40). Allocate (10,25,40) units to cell (1,4).

						SUPPLY
	7	10	14	8	(10,25,40) 0	(25,30,35)
	7	12	12	6	0	(25,40,55)
	5	8	15	9	0	(40,50,60)
DEMAND	(10,20,30)	(15,20,25)	(15,25,35)	(10,30,50)	(10,25,40)	

Proceeding in this way we get the final iteration,

FINAL ITERATION

Check whether all the cells are allocated. If not, repeat the process. Thus the fuzzy initial basic feasible solution is

(-15,5,25) 7	10	14	8	(10,25,40) 0
(-25,10,45) 7	12	12	(10,30,50) 6	0
(-60,5,70) 5	(15,20,25) 8	(-55,25,105) 15	9	0

Fuzzy Transportation Cost

Z = (-15,5,25)× 7 +(10,25,40) × 0 + (−25,10,45) × 7 + (10,30,50) × 6 +
(−60,5,70) × 5 + (15,20,25) × 8 + (−55,25,105) × 15
= (-105,35,175) + (0,0,0) + (-175,70,315) +(60,180,300) +(-300,25,350) + (120,160,200) + (-825,375,1575)
= (-1225,845,2915)

Z = 845

Defuzzified value is 845

ROW MINIMA METHOD

FIRST ITERATION

Select the least expensive in first row for allocation. In the table below, the minimum entry is "(0)" in cell (1,4). Allocation for cell (1,4) = min(3,5,7),(0,1,2) = (0,1,2). Allocate (0,1,2) units to cell (1,4)

					SUPPLY
	2	7	14	(0,1,2) 0	(3,5,7)
	3	3	1	0	(5,8,11)
	5	4	7	0	(4,710,)
	1	6	2	0	(10,15,20)
DEMAND	(4,7,10)	(5,9,13)	(13,18,23)	(0,1,2)	

Proceeding in this way we get the final iteration.

FINAL ITERATION

Check whether all the cells are allocated. If not, repeat the process. Thus the fuzzy initial basic feasible solution is:

(1,4,7) 2	7	14	(0,1,2) 0
3	3	(5,8,11) 1	0
5	(4,7,10) 4	7	0
(-3,3,9) 1	(-17,2,21) 6	(2,10,18) 2	0

Fuzzy Transportation Cost

$$Z=(1,4,7 \times 2 + (0,1,2) \times 0 + (5,8,11) \times 1 + (4,7,10) \times 4 + (-3,3,9) \times 1 + (-17,2,21) \times 6 + (2,10,18) \times 2$$

= (2,8,14) + (0,0,0) + (5,8,11) + (16,28,40) + (-3,3,9) + (-102,12,126) + (4,20,36)
= (-78,79,236)
Z = 79
Defuzzified value is 79

FUZZY TRANSPORTATION PROBLEM WITH FUZZY COST
PROBLEM 4.1

A company has three factories and five warehouses. The commodities are transported from the factories to the warehouses, which are located at various distances from the factories. The warehouse requirements are d_1= (20),d_2= (20),d_3= (25),d_4= (30),d_5= (25). The requirements of each factory are given by, s_1= (30), s_2=(40), s_3=(50). The table below shows the costs of transportation from factories to warehouses.

						SUPPLY
	(4,7,10)	(8,10,12)	(10,14,18)	(6,8,10)	(0,0,0)	30
	(4,7,10)	(4,12,20)	(4,12,20)	(5,6,7)	(0,0,0)	40
	(1,5,9)	(6,8,10)	(10,15,20)	(6,9,12)	(0,0,0)	50
DEMAND	20	20	25	30	25	

4.1.1 NORTH WEST CORNER RULE METHOD

FIRST ITERATION

Select the north-west cell for allocation. In the table below, the minimum entry is "(4,7,10)" in cell (1,1). Allocation for cell (1,1) = min(20,30) = (20). Allocate (20) units to cell (1,1).

						SUPPLY
	20 (4,7,10)	(8,10,12)	(10,14,18)	(6,8,10)	(0,0,0)	30
	(4,7,10)	(4,12,20)	(4,12,20)	(5,6,7)	(0,0,0)	40
	(1,5,9)	(6,8,10)	(10,15,20)	(6,9,12)	(0,0,0)	50
DEMAND	20	20	25	30	25	

Proceeding in this way we get the final iteration.

FINAL ITERATION

Check whether all the cells are allocated. If not, repeat the process. Thus the fuzzy initial basic feasible solution is,

20 (4,7,10)	10 (8,10,12)	(10,14,18)	(6,8,10)	(0,0,0)

(4,7,10)	10	25	5	(0,0,0)
	(4,12,20)	(4,12,20)	(5,6,7)	
(1,5,9)	(6,8,10)	(10,15,20)	25	25
			(6,9,12)	(0,0,0)

Fuzzy Transportation Cost

$$Z=20\times(4,7,10)+10\times(8,10,12)+10\times(4,12,20)+25\times(4,12,20)+5\times(5,67)+25\times(6,9,12)+25\times(0,0,0)$$

= (80,140,200)+(80,100,120)+(40,120,200)+(100,300,500)+(25,30,35)+(150,225,300)+(0,0,0)

= (475,915,1355)

Z = 915

Defuzzified value is 915

4.1.2 ROW MINIMA METHOD

FIRST ITERATION

Select the least expensive in first row for allocation. In the table below, the minimum entry is "(0,0,0)" in cell (1,4). Allocation for cell (1,4) = min(30,25) = (25). Allocate (25) units to cell (1,4).

						SUPPLY
	5	(8,10,12)	(10,14,18)	(6,8,10)	(0,0,0)	30
	(4,7,10)					
	(4,7,10)	(4,12,20)	(4,12,20)	(5,6,7)	(0,0,0)	40
	(1,5,9)	(6,8,10)	(10,15,20)	(6,9,12)	(0,0,0)	50
DEMAND	20	20	25	30	25	

Proceeding in this way we get the final iteration.

FINAL ITERATION

Check whether all the cells are allocated. If not, repeat the process. Thus the fuzzy initial basic feasible solution is:

5	(8,10,12)	(10,14,18)	(6,8,10)	25
(4,7,10)				(0,0,0)

10			30	(0,0,0)
(4,7,10)	(4,12,20)	(4,12,20)	(5,6,7)	
5	20	25	(6,9,12)	(0,0,0)
(1,5,9)	(6,8,10)	(10,15,20)		

Fuzzy Transportation Cost

Z = 5× (4,7,10) + 25 × (0,0,0) +10× (4,7,10) + 30 × (5,6,7) + 5 × (1,5,9) + 20 × (6,8,10) + 25 × (10,15,20)

= (20,35,50)+(0,0,0)+(40,70,100)+(150,180,210)+(5,25,45)+(120,160,200)+ (250,375,500)

= (585,845,1105)

Z = 845

Defuzzified value is 845

FUZZY TRANSPORTATION PROBLEM WITH FUZZY COST AND FUZZY DEMAND AND SUPPLY

PROBLEM 5.1

A company has three factories and five warehouses. The commodities are transported from the factories to the warehouses, which are located at various distances from the factories. The warehouse requirements are d_1= (10,20,30), d_2=(15,20,25), d_3=(15,25,35), d_4=(10,30,50) ,d_5=(10,25,40). The requirements of each factory are given by, s_1=(25,30,35), s_2=(25,40,55), s_3=(40,50,60). The table below shows the costs of transportation from factories to warehouses.

						SUPPLY
	(4,7,10)	(8,10,12)	(10,14,18)	(6,8,10)	(0,0,0)	(25,30,35)
	(4,7,10)	(4,12,20)	(4,12,20)	(5,6,7)	(0,0,0)	(25,40,55)
	(1,5,9)	(6,8,10)	(10,15,20)	(6,9,12)	(0,0,0)	(40,50,60)
DEMAND	(10,20,30)	(15,20,25)	(15,25,35)	(10,30,50)	(10,25,40)	

5.1.1 NORTH WEST CORNER RULE METHOD

FIRST ITERATION

Select the north-west cell for allocation. In the table below, the minimum entry is "(4,7,10)" in cell (1,1). Allocation for cell (1,1) = min(10,20,30),(25,30,35) = (10,20,30). Allocate (10,20,30) units to cell (1,1).

						SUPPLY
	(10,20,30) (4,7,10)	(8,10,12)	(10,14,18)	(6,8,10)	(0,0,0)	(25,30,35)
	(4,7,10)	(4,12,20)	(4,12,20)	(5,6,7)	(0,0,0)	(25,40,55)
	(1,5,9)	(6,8,10)	(10,15,20)	(6,9,12)	(0,0,0)	(40,50,60)
DEMAND	(10,20,30)	(15,20,25)	(15,25,35)	(10,30,50)	(10,25,40)	

Proceeding in this way we get the final iteration.

FINAL ITERATION

Check whether all the cells are allocated. If not, repeat the process. Thus the fuzzy initial basic feasible solution is

(10,20,30) (4,7,10)	(-5,10,25) (8,10,12)	(10,14,18)	(6,8,10)	(0,0,0)
(4,7,10)	(-10,10,30) (4,12,20)	(15,25,35) (4,12,20)	(-40,5,50) (5,6,7)	(0,0,0)
(1,5,9)	(6,8,10)	(10,15,20)	(-40,25,90) (6,9,12)	(-50,25,100) (0,0,0)

Fuzzy Transportation Cost

$$Z = (10,20,30)\times(4,7,10)+(-5,10,25)\times(8,10,12)+(-10,10,30\times(4,12,20)+(15,25,35)\times(4,12,20)+(-40,5,50)\times(5,6,7)+(-40,25,90)\times(6,9,12)+(-50,25,100)\times(0,0,0)$$

$$=20\times 7+10\times 10+10\times 12+25\times 12+5\times 6+25\times 9+25\times 0$$

=140+100+120+300+30+225+0

Z =915

Defuzzified value is 915

CONCLUSION

The methods discussed namely **NORTH WEST CORNER RULE, ROW MINIMA METHOD, COLUMN MINIMA METHOD, MATRIX MINIMA METHOD, VOGEL'S APROXIMATION METHOD** has the major advantage, that it is very easy to understand. The procedure for the solution is illustrated with the numerical examples. Further, comparative study among the algorithms is established by means of simple problems.

Transportation models have wide applications in logistics and supply chain, with the purpose of reducing the cost. Previous studies have devised solution procedures for transportation problems. In real world applications, the parameters in the transportation problem may not be known, precisely due to uncontrollable factors. If the obtained results are crisp values, then the transportation problem might loss some useful information. Since the objective value is expressed by the membership function, rather than by a crisp value, more information is provided for making decisions.

The project entitled "Fuzzy Transportation Problem" is the consolidation of different notions that enrich the realm of the transportation problems in fuzzy environment. More particularly, the project contemplates different transportation algorithms pertaining to the fuzzy transportation problems. Better solution procedures have been provided for solving fuzzy transportation problems in the fuzzy environment yielding from the traditional way of solving.

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