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HARMONIC MEAN LABELLING OF SUBDIVISION AND RELATED GRAPHS

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ABSTRACT

In this paper, we contribute some new results for Heronian Mean labeling of graphs. We have already proved that subdivisions of Heronian Mean Graphs are again Heronian Mean Graphs. We use some more standard graphs to derive the results of Heronian Mean labeling for subdivision of graphs. A graph G = (V, E) with p vertices and q edges is said to be a Harmonic mean graph if it is possible to label the vertices x∈V with distinct labels f(x) from 1, 2, …, q+1 in such a way that when each edge e =uv is labeled with f(e = uv) = ⎸⎸⏋⎸⎸⎾ ( ) + ( ) 2 ( ) ( ) f u f v f u f v (or) ⎸⏌⎸⎸⎿⎸ ( ) + ( ) 2 ( ) ( ) f u f v f u f v , then the edge labels are distinct. In this case f is called Harmonic mean labeling of G. In this paper we prove the Harmonic mean labeling behavior for some special graphs.

Keywords: Graph, Harmonic mean graph, path, comb, kite, Ladder, Crown.

1. INTRODUCTION

In this paper, we consider only finite, simple and undirected graphs. Let G(V,E) be a graph with p vertices and q edges. For notations and terminology we follow [1]. In a graph G, the subdivision of an edge uv is the process of deleting the edge uv and introducing a new vertex w and the new edges uw and vw. If every edge of G is subdivided exactly once, then the resultant graph is denoted by S(G) and is called the subdivision graph of G. For example, a star $K_{1,5}$ and its subdivision graph S($K_{1,5}$).
The union of two graphs G_1 and G_2 is a graph $G_1 \cup G_2$ with vertex set $V(G_1 \cup G_2)=V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2)=E(G_1) \cup E(G_2)$.$mG$ denotes the disjoint union of m copies of G. Bistar $B_{n,m}$ is the graph obtained from $K_{1,n} \cup K_{1,m}$ by joining the central vertices of $K_{1,n}$ and $K_{1,m}$ by means of an edge. The newly added edge is called the central edge of $B_{n,m}$.

An assignment $f:V(G) \to \{0,1,2,\ldots,q\}$ is called a mean labelling if whenever each edge $e=uv$ is labeled with $[\frac{2f(u)f(v)}{2}]$ if $f(u)+f(v)$ is even and $[\frac{f(u)+f(v)+1}{2}]$ if f(u)+f(v) is odd ,then the resulting edge labels are all distinct.Any graph that admits a mean labelling is called a **mean graph**. Many results on mean labelling have been proved in [4] and [5]. In a similar way, [6] have introduced the concept of harmonic mean labelling of a graph.

An assignment $f:V(G) \to \{1,2,\ldots,q+1\}$ is called a **harmonic mean labelling.** If whenever each edge $e=uv$ is labeled with $\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$then the edge labels are distinct. Any graph that admits a harmonic mean labelling is called a **harmonic mean graph.**
More results on harmonic mean labelling have been proved in [7].A well collection of results on graph labelling has been done in the survey [2].

In this paper, we establish the harmonic mean labelling of some standard graphs like subdivision of star ($K_{1,}$), subdivision of bistar ($B_{n,}$),the disconnected graph $S(K_{1,n}) \cup kC_m$ etc.

By a graph, we mean a finite undirected graph without loops or multiple edges. The vertex set is denoted by V(G) and the edge set is denoted by E(G).

A cycle of length n is C_n and a path of length n is denoted by P_n. For all other standard terminology and notations we follow Harary [1].

We introduce Harmonic mean labelling of graphs in [3] and studied their behavior in [4], [5] and [6]. In this paper, we investigate Harmonic mean labelling for some special graphs. The definition and other information which are useful for the present investigation are given below.

2. PRELIMINARY DEFINITION

Definition 1.1:

A graph G= (V,E) with p vertices and q edges is called a Harmonic mean graph if it is possible to label the vertices x∈V with distinct lables from 1,2…q+1 in such a way that wheh edge e=uv is labeled with $f(e=uv)=\left[\frac{2f(u)f(v)}{f(u)f(v)}\right]$ then the edge labels are distinct. In this case, f is called a **Harmonic mean labelling** of G.

Definition 1.2:

Let v be a vertex of a graph G. Then the **duplication** of v is a graph G(v) obtained from G by adding a new vertex v'with N(v′)=N(v).

Definition 1.3:

Let e=uv be an edge of G. Then **duplication of an edge** e=uv is a graph G(uv) obtained from G by adding a new edge u′v′ such that N(u′)=N(u)∪{v′}-{v} and N(v′)= N(v)∪{u′}-{u}

Definition 1.4:

Consider two copies of C_n, connect a vertex of first coy to a vertex of second copy with a new edge, the new graph obtained is called **joint sum** of C_n.

Definition 1.5:

Let u and v be two distinct vertices of a graph G. A new graph $G_1$is constructed by identifying two vertices u and v by a single vertex w is such that every edge which was incident with either u or v in G is not incident with w in G.

3. HARMONIC MEAN LABELLING OF SOME CYCLE RELATED GRAPHS

Theorem 2.1:

The graph obtained by duplicating an arbitrary edge in cyclen is a Harmonic mean graph.

Proof:

Let $C_n= v_1v_2\ldots v_nv_1$ be the cycle.

Let $e'= u_1'u_2'$ be the duplicated edge of $e= u_1u_2$

Now we define $f: V(G(u_1u_2)) \rightarrow \{1,2\ldots q+1\}$ by

$f(u_1) = 1$

$f(u_2) = 2$

$f(u_3) = 3$

$f(u_i) = i+1,\ 4 \leq i \leq n$

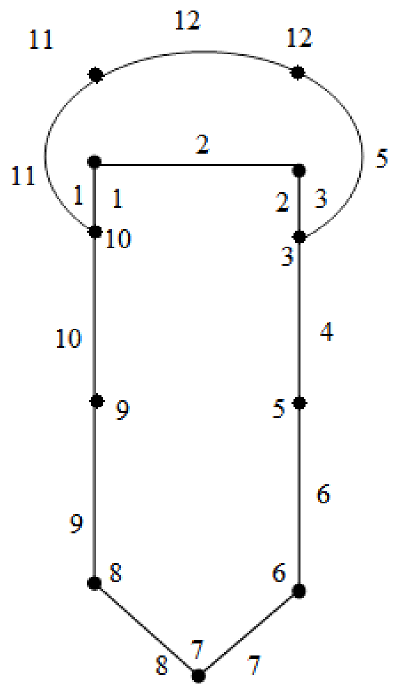
$f(u_1') = n$

$f(u_2') = n+1$

Hence f is a Harmonic mean labelling of duplicated graph $G(u_1u_2)$

Example 2.2:

The following is the harmonic mean labelling of $C_9(u_1u_2)$



HARMONIC MEAN LABELLING OF SUBDIVISION AND RELATED GRAPHS

Theorem 3.1

The disconnected graph $(K_{1,n}) \cup kC_m$ is a harmonic mean graph for

$1 \le n \le 5$, $m \ge 3$ and $k \ge 0$

Proof

Let $V(S(K_{1,n}) \cup kC_m)=\{v;\, u_1,u_2,\ldots,u_n; v_1,v_2,\ldots,v_n; w_{11},w_{12},\ldots,w_{1m};\, w_{21},w_{22},\ldots,w_{2m};\ldots w_{k1},w_{k2},\ldots,w_{km}\}$ and

$E(S(K_{1,n}) \cup kC_m)=\{vu_i, u_iv_i | 1\le i\le n\}\cup[U_{i=1}^{k}((U_{j=1}^{M-1}\{W_{ij}\, W_{ij+1}\})U\{W_{im}W_{i1}\}]$.

Here $p=2n+km+1$ and $q=2n+km$.

Define a function $f: V(S(K_{1,n}) \cup kC_m) \rightarrow \{1,2,\ldots,q+1\}$ by

$f(v) = 2n+1$;

$f(u_i) = n+i$, $1 \le i \le n$;

$f(v_i) = n+1-i$, $1 \le i \le n$ and

$f(w_{ij}) = (2n+1)+(i-1)m+j, 1 \le i \le k, 1 \le j \le m$.

Then the induced edge labels of $(K_{1,})$ are given below:

$f^*(u_nv_n) = 1$;

$f^*(u_iv_i) = n+1+i, 1 \le i \le n$;

$f^*(u_iv_i) = n+2-i, 1 \le i \le n-1;$

and the set of all edge labels of kC_m is $\{2(n+1), 2n+3, \ldots, 2n+k\text{m}+1\}$.

Therefore the set of all edge labels of $(K_{1,}) \cup kC_m$ is

$\{1, 3, 4 \ldots .2n+k\text{m}+1\}$.

Hence $(K_{1,}) \cup kC_m$ is a harmonic mean graph for $1 \le n \le 5, \ge 0$ and $\text{m} \ge 3$.

Hence the theorem.

Theorem 3.2

The disconnected graph $(B_{3,4}) \cup kC_m$ is a harmonic mean graph for $k \ge 0$ and $\text{m} \ge 3$.

Proof:

Let $V(S(B_{3,4}) \cup kC_m) = \{u; u_1, u_2, \ldots, u_3; v_1, v_2, \ldots, v_3; y; x; x_1, x_2, \ldots, x_4;$

$y_1, y_2, \ldots y_4; w_{11}, w_{12}, \ldots, w_{1m}; w_{21}, w_{22}, \ldots, w_{2m}; \ldots w_{k1}, w_{k2}, \ldots w_{km}\}$ .and

$E(S(B_{n1,n2}) \cup kC_m) = \{ u_iv_i, uv_i, uy, yx, xx, x_jy_j | \; 1 \le i \le 3 \; , \; 1 \le j \le 4\} \cup$

$$[U_{i=1}^{K} \; ((U_{j=1}^{m-1}\{w_{ij}w_{ij+1}) \cup \{w_{im}\, w_{i1}\})].$$

Here p $=17+k$m and q $=16+k$m.

Define a function f $:((_{3,4}) \cup kC_m) \rightarrow \{1, 2, \ldots, q+1\}$ by

f$(u) = 7;$

f$(u_i) = 3+ , 1 \le i \le 3;$

f$(v_i) = 4-i \; , 1 \le i \le 3;$

f$(y) = 8;$

f$(x) = 17;$

f$(x_j) = 12+j, 1 \le j \le 4;$

f$(y_j) = 8+j, 1 \le j \le 4$ and

f$(w_{ij}) = 17+(i-1)\text{m}+j, 1 \le i \le k, 1 \le j \le \text{m}$.

Then the induced edge labels of $(B_{3,4})$ are given below:

$f^*(u_nv_n) = 1;$

$f^* \; (u_i\text{v}_i) = 4+i, 1 \le i \le 3;$

$f^*(u_iv_i) = 5-i, 1 \le i \le \; 2;$

$f^*(u\text{v}) = 8;$

$f^*(uy) = 9;$

$f^*(\text{x}_j) \; = 13+j, 1 \le j \le 4;$

$f^*(\text{x}_j\text{y}_j) = 9+j, 1 \le j \le 4;$

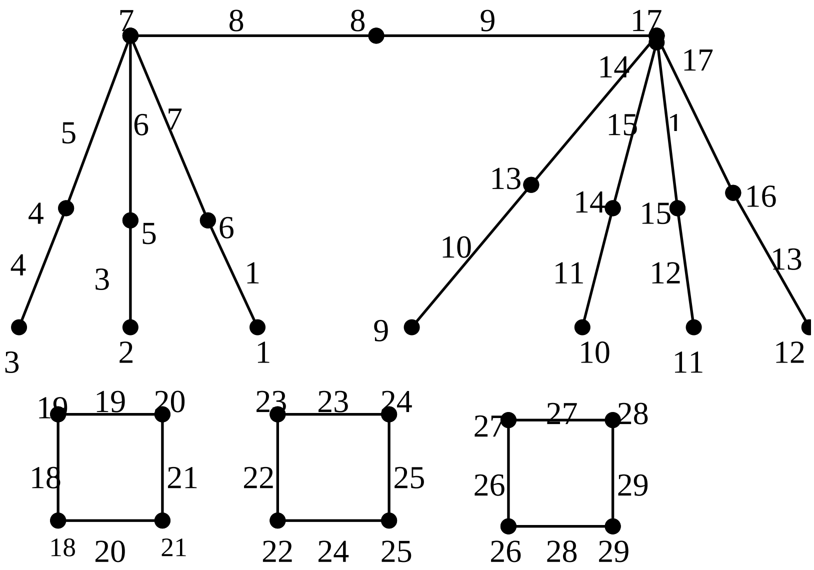
and the set of all edge labels of kC_m is $\{18, 19, \ldots, 17+k\text{m}\}$.

Therefore the set of all edge labels of $S(B_{3,4}) \cup kC_m$ is $\{ 1, 3, 4, \ldots .17+k\text{m} \}$.

Hence $(B_{3,4}) \cup kC_m$ is a harmonic mean graph for $k \geq 0$ and $m \geq 3$.

Hence the theorem.

As an example harmonic mean labelling of $(B_{3,4}) \cup 3C_4$



Theorem 3.3

The graph $P_n^* \cup kC_m$ is a harmonic mean graph for $k \geq 0$, $m \geq 3$ and $n \geq 2$.

Proof:

Let $V(P_n^* \cup kC_m) = \{v_1, v_2, \ldots, v_n; u_1, u_2, \ldots, u_{n-1}; w_{11}, w_{12}, \ldots, w_{1m}; w_{21}, w_{22}, \ldots, w_{2m}; w_{k1}, w_{k2}, \ldots, w_{km}\}$and

$E(P_n^*) = \{v_i v_{i+1};\ v_i u_i;\ u_i v_{i+1} \mid 1 \leq i \leq n-1\ ;\ u_i u_{i+1} \mid 1 \leq i \leq\ n-2\} \cup [U_{i=1}^{k}((U_{j=1}^{m-1}\ \{w_{ij}\ w_{ij+1}\}) \cup \{w_{im} w_{i1}\})]$.

Here $p = 2n+km-1$ and $q = 4n-5+km$.

Define a function f $:(^*_n \cup kC_m) \rightarrow \{1, 2, \ldots, q+1\}$ by

$f(v_n) = 4(n-1);$

$f(v_i) = 4i-3, 1 \leq i \leq n-1;$

$f(u_i) = 4i-1, 1 \leq i \leq n-1$ and

$f(w_{ij}) = 4(n-1)+(i-1)m+j, 1 \leq i \leq k, 1 \leq j \leq m.$

Then the induced edge labels of P_n^* are given below:

$f^*(v_i v_{i+1}) = 4i-2,\ 1 \leq i \leq n-1;$

$f^*(u_i u_{i+1}) = 4i,\quad 1 \leq i \leq n-2;$

$f^*(v_i u_i) = 4i-3,\ 1 \leq i \leq n-1;$

$f^*(u_i v_{i+1}) = 4i-1,\ 1 \leq i \leq n-1;$

and the set of all edge labels of kC_m is $\{4n-3, 4n-4, \ldots, 4(n-1)+km\}$.

Therefore the set of all edge labels of $P_n^* \cup kC_m$ is

$\{1, 2, 3 \ldots, 4(n-1)+km\}$.

Hence $P_n^* \cup kC_m$ is a harmonic mean graph for $k \geq 0$, $m \geq 3$ and $n \geq 2$.

Hence the theorem.

K-HARMONIC MEAN LABELLING OF SOME GRAPHS

Theorem

The path P_n a k-harmonic mean graph for all k and $n \geq 2$.

Proof

Let $V(P_n) = \{v_i; 1 \leq i \leq n\}$ and

$E(P_n) = \{e_i = v_i, v_{i+1}; 1 \leq i \leq n-1\}$

Define a function $f : V(P_n) \rightarrow \{k, k+1, k+2, ..., k+q\}$ by

$f(v_i) = k + i - 1, \forall 1 \leq i \leq n$

Then the induced edge labels are

$f^*(e_i) = k + i - 1, \forall 1 \leq i \leq n-1$

The above defined function f provides k-harmonic mean labelling of the graph.

Hence P_n is a k-harmonic mean graph.

Theorem:

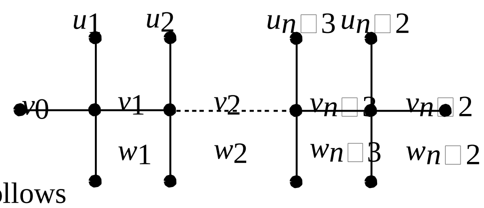
The Twig graph T_n is k-harmonic mean graph for all $n \geq 3$.

Proof:

Let $V(T_n) = \{v_i; 0 \leq i \leq n-1, u_i, w_i; 1 \leq i \leq n-2$ and

$E(T_n) = \{v_i u_i, v_i w_i; 1 \leq i \leq n-2, v_i v_{i+1}; 0 \leq i \leq n-2$

The ordinary labelling is



First we label the vertices as follows

Define the function $f : V(T_n) \rightarrow \{k, k+2, k+2, ..., k+q\}$ by

$f(v_0) = k$

$f(v_i) = k + 3i - 2,$ for $1 \leq i \leq n-1$

$f(w_i) = k + 3i - 1,$ for $1 \leq i \leq n-2$

$f(u_i) = k + 3i,$ for $1 \leq i \leq n-2$

Then the induced edge labels are

$f^*(v_i v_{i+1}) = k + 3i,$ for $0 \leq i \leq n-2$

$f^*(v_i u_i) = k + 3i - 1,$ for $1 \leq i \leq n-2$

$f^*(v_i w_i) = k + 3i - 2,$ for $1 \leq i \leq n-2$

The above defined function f provides k-harmonic mean labelling of the graph.

CONCLUSION

In this paper we discuss Harmonic mean labelling behaviour of some cycle related graphs such as duplication, joint sum of the cycle and identification of cycle. Also we investigate Harmonic mean labelling behaviour of Alternate Triangular Snake A(Tn), Alternate Quadrilateral Snake A(Qn).In this paper, we establish harmonic mean labels of some well known subdivision graphs and some disconnected graphs.In this paper we prove the Harmonic mean labelling behaviour for some special graphs. In this paper, we establish the harmonic mean labelling of some standard graphs like subdivision of star ($K_{1,}$), subdivision of bistar ($B_{n,}$),the disconnected graph $S(K_{1,n}) \cup kC_m$.

BIBLOGRAPHY

1) Harary.F., 1988, Graph theory, Narosa Publishing House, New Delhi.
2) Somasundaram.S., and Ponraj R., 2003, Mean labelling of graphs National Academy of Science Letters vol.26, p.210-213
3) Somasundaram S., Ponraj R., and Sandhya S.S., Harmonic mean labelling of graphs communicated.
4) Sandhya S.S., Somasundaram S., and Ponraj R., Some Results on Harmonic Mean Graphs, International journal of Contemporary Mathematical Sciences 7(4) (2012), 197-208.
5) Sandhya S.S., Somasundaram S., and Ponraj R., Some More Results on Harmonic Mean Graphs Journal of Mathematics Research 4(1) (2012) 21-29.
6) Sandhya S.S., Somasundaram S., and Ponraj R. , Harmonic Mean Labelling of Some Cycle Related Graphs, International Journal of Mathematics Analysis vol.6, 2012. No.40 1997-2005

